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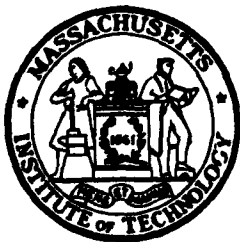
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**HIERARCHICAL PRODUCTION PLANNING:  
A TWO STAGE SYSTEM**

by  
**GABRIEL R. BITRAN  
ELIZABETH A. HAAS  
and  
ARNOLDO C. HAX**

**Technical Report No. 179  
OPERATIONS RESEARCH CENTER**

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**May 1980**

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report No. 179	2. GOVT AC. ESSION NO. AD-A086152	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) HIERARCHICAL PRODUCTION PLANNING: A TWO STAGE SYSTEM.	5. TYPE OF REPORT & PERIOD COVERED Technical Report May 1980	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Gabriel R. Bitran Elizabeth A. Haas Arnoldo C. Hax	8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0556	
9. PERFORMING ORGANIZATION NAME AND ADDRESS M.I.T. Operations Research Center 77 Massachusetts Avenue Cambridge, MA 02139	10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS NR 347-027	
11. CONTROLLING OFFICE NAME AND ADDRESS O.R. Branch, ONR Navy Dept. 800 North Quincy Street Arlington, VA 22217	12. REPORT DATE May 1980	13. NUMBER OF PAGES 33 pages
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 34 71-111	15. SECURITY CLASS. (of this report)	15a. DECLASSIFICATION DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Releasable without limitation on dissemination.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Two Stage Processes Fabrication and Assembly Aggregate Production Planning Production Planning Disaggregation		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) See page ii.		

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S/N 0102-LF-014-6601

Unclassified

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Technical Report No. 179

Work Performed Under  
Contract N00014-75-C-0556, Office of Naval Research  
Multilevel Logistics Organization Models  
Project No. NR 347-027

Operations Research Center  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139

May 1980

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## FOREWORD

The Operations Research Center at the Massachusetts Institute of Technology is an interdepartmental activity devoted to graduate education and research in the field of operations research. The work of the Center is supported, in part, by government contracts and grants. The work reported herein was supported by the Office of Naval Research under Contract No. N00014-75-C-0556. One of the authors, Elizabeth A. Haas, would also like to acknowledge the support of the Operations Management Practice Group of McKinsey and Company.

Richard C. Larson  
Jeremy F. Shapiro

Co-Directors

## ABSTRACT

→ This paper presents a hierarchical approach to plan and schedule production in a manufacturing environment that can be modeled as a two stage process. A conceptual framework for this approach is described. The specific mathematical models proposed for the various hierarchical levels are discussed. The methodology is evaluated in an actual setting. The performance of the hierarchical system is contrasted with an MRP design. Encouraging results are reported. ←

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## 1. INTRODUCTION

The management of the production process involves complex choices among a large number of alternatives. These choices have to be made by trading off conflicting objectives in the presence of financial, technological, and marketing constraints. Since its early developments, operations research found the field of production planning a most fruitful area of application.

Initial operations research contributions to production planning tended to address individual sub-problems. More recently, attention has been focused on the design of integrative model-based approaches to support the overall spectrum of production management decisions. Hierarchical planning systems represent one methodology suggested to deal with the entirety of production management issues. In a recent publication [1] the authors have addressed the characteristics of those systems for single-stage production processes. The logic of single-stage hierarchical planning systems will not be presented here. For an extensive coverage of these subjects, the reader is referred to [2],[6],[7].

The objective of this paper is to discuss extensions of hierarchical production planning (HPP) systems to support two-stage production processes. This is an important area of concern since many manufacturing environments can be described in terms of two-stage processes. The most relevant of such environments are those involving fabrication and assembly operations, where activities have to be planned in a coordinated way. Figure 1.1 illustrates a simplified representation of a two-stage setting. A conceptual overview of a two-stage HPP system is given in Figure 1.2. The essence of the approach can be summarized as follows:

- First, individual parts and finished products are grouped into aggregate parts and aggregate finished products.

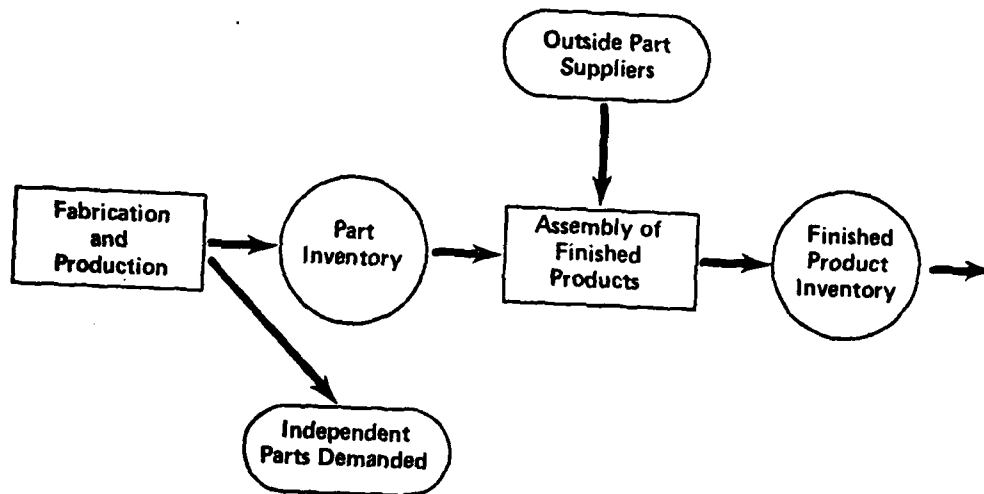


Figure 1.1: The Two-Stage Setting

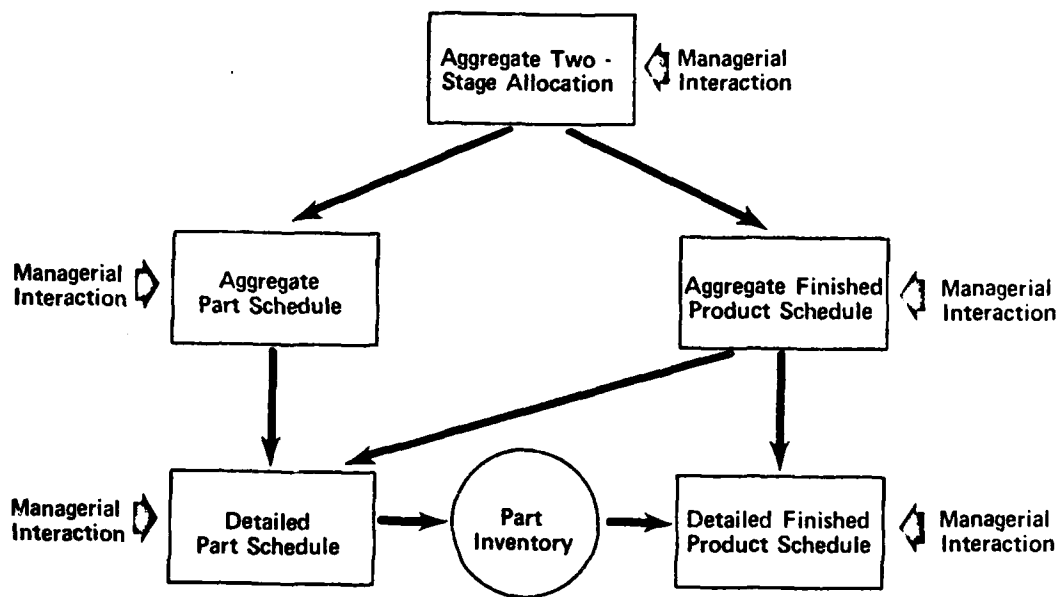


Figure 1.2: A Conceptual Overview of a Hierarchical Production Planning System for a Fabrication and Assembly Process



- Second, an aggregate model is used to schedule the corresponding production quantities for those aggregate parts and finished products. The model addresses this decision jointly, thus guaranteeing the appropriate coordination of the two-stage process.

- Third, the aggregate part production and finished product production plans are disaggregated to determine the detailed schedules for individual parts and finished products.

- Fourth, a reconciliation of possible differences at the detail level is performed via part inventories.

We favor an aggregate allocation approach at the higher level of the hierarchical system to avoid the massive data manipulation, computational complexities, and forecasting inaccuracies that would be imposed by a detailed allocation model at that level. Furthermore, we do not believe that a detailed formulation is necessary to capture the essential trade-offs and constraints inherent in the production planning process at a tactical level. Finally, the HPP approach allows for effective managerial interaction at all levels of the decision making process, as explained in [1].

The overall hierarchical design can be viewed as an alternative to Material Requirements Planning (MRP), the most widely used design philosophy to deal with two-stage production planning issues (see [10],[12],[13]). However, some elements of the hierarchical framework can also be constructively used to enhance an MRP system.

In sections 2 and 3 we describe the characteristics of the proposed HPP system. A brief description of MRP and computational results contrasting HPP with MRP are presented in section 4. Conclusions are given in section 5.

## 2. TWO-STAGE HIERARCHICAL PLANNING (HPP) - THE AGGREGATE PLANNING MODEL

The highest level of planning in the hierarchical approach determines production schedules for aggregate parts and aggregate finished products (see Figure 1.2). Thus, the first design decision to be made concerns the way in which individual parts and finished products are to be aggregated.

The criterion for the aggregation of finished products follows quite closely the one adopted for the single-stage hierarchical planning system [1]. The aggregation used for finished products in the two-stage setting is as follows:

Product Items: are end finished products delivered to customers.

Product Types: are groups of finished product items having similar direct production costs (excluding labor), holding cost per unit per period, productivities (number of units that can be produced per unit of time), and seasonalities.

Product Families: are groups of finished product items sharing a major setup cost and requiring an identical number of the same parts.

The aggregation criterion for parts recognizes only one level of aggregation:

Part Items: are individual parts either required as a component to a product item or having an independent demand as a service or spare part.

Part Types: are groups of part items having similar direct production costs, holding costs per part per period, and productivities. Part items also share common fabrication facilities. For parts, two levels of aggregation was sufficient as no two items shared a setup cost.

This aggregation of finished products and parts is applicable to many industrial settings encountered in practice and was inspired by a

real-life situation that will be discussed in section 4. This framework can be adjusted to fit several variants of the proposed aggregation structure and should not be seen as a limitation of the hierarchical methodology to be presented.

## 2.1 Aggregate Production Planning for Product Types and Part Types

The aggregate two-stage allocation model introduced here is formulated as a linear program. We have chosen this representation because in the vast majority of practical instances, production allocation decisions lend themselves quite naturally to be treated by linear programming. However, any of the aggregate production models suggested in the literature ([5] and [8]) could have been used as long as they provide an acceptable formulation of the process being considered.

### The Aggregate Two-Stage Linear Programming Model

Problem (P)

$$\begin{aligned}
 & \text{minimize} \quad \sum_{t=1}^T \left[ \sum_{i=1}^I (h_{it} I_{it} + r_t R_{it} + o_t O_{it}) + \right. \\
 & \quad \left. \sum_{k=1}^K (\hat{h}_{kt} \hat{I}_{kt} + \hat{r}_{kt} \hat{R}_{kt} + \hat{o}_{kt} \hat{O}_{kt}) \right] \\
 & \text{subject to} \quad I_{it-1} + m_i (R_{it} + O_{it}) + I_{it} = d_{it} \\
 & \quad \quad \quad i=1,2,\dots,I; t=1,2,\dots,T \quad (2.1) \\
 & \quad \quad \quad \sum_{i=1}^I R_{it} \leq (rm)_t \quad t=1,2,\dots,T \\
 & \quad \quad \quad \sum_{i=1}^I O_{it} \leq (om)_t \quad t=1,2,\dots,T \\
 & \quad \quad \quad ss_{it} \leq I_{it} \leq os_{it} \quad i=1,2,\dots,I; t=1,2,\dots,T \\
 & \quad \quad \quad \sum_{k=1}^K \hat{R}_{kt} \leq (\hat{rm})_t \quad t=1,2,\dots,T
 \end{aligned}$$

$$\sum_{k=1}^K \hat{O}_{kt} \leq (\hat{om})_t \quad t=1,2,\dots,T$$

$$\hat{ss}_{kt} \leq \hat{I}_{kt} \leq \hat{os}_{kt} \quad k=1,2,\dots,K; t=1,2,\dots,T$$

$$\hat{I}_{kt-1} + \hat{m}_k(\hat{R}_{kt} + \hat{O}_{kt}) - \hat{I}_{kt} =$$

$$\sum_{i=1}^I f_{ik} m_i (R_{it+L} + O_{it+L}) \quad k=1,2,\dots,K; t=1,2,\dots,T-L$$

(2.2)

$$R_{it}, O_{it}, I_{it}, \hat{R}_{kt}, \hat{O}_{kt}, \text{ and } \hat{I}_{kt} \geq 0$$

$i=1,\dots,I; k=1,2,\dots,K;$   
 $t=1,2,\dots,T.$

The indices  $i$ ,  $k$ , and  $t$  represent, respectively, product types, part types, and time periods. The parameters  $h_{it}$ ,  $r_t$ ,  $o_t$ ,  $m_i$ ,  $d_{it}$ ,  $(rm)_t$ ,  $(om)_t$ ,  $ss_{it}$ , and  $os_{it}$  denote, respectively, the cost of holding one unit of inventory of product type  $i$  from period  $t$  to period  $t+1$ , the cost of one hour of regular labor in period  $t$ , the cost of one hour of overtime in period  $t$ , the productivity of product type  $i$ , the effective demand of units of product type  $i$  in period  $t$ , the number of regular labor hours in period  $t$ , the number of overtime hours in period  $t$ , the safety stock of product type  $i$  in period  $t$ , and the overstock limit of product type  $i$  in period  $t$ . The parameters with a " $\hat{\phantom{x}}$ " have the same meaning for part types.

The number of units of part type  $k$  required per unit of product type  $i$  is represented by  $f_{ik}$ . This parameter is discussed later in this section. The variables  $R_{it}$ ,  $O_{it}$ , and  $I_{it}$  denote the number of hours of regular labor time, the number of hours of overtime, and the number of units in inventory for product type  $i$  in period  $t$ . The variables with a " $\hat{\phantom{x}}$ " have the same meaning for part type  $k$  in period  $t$ . The fabrication lead time of parts is denoted by  $L$ . The labor unit cost for part types has been

assumed to be a function of each part type, while the labor unit cost for assembly,  $r_t$  and  $o_t$ , are taken equal for all product types. These assumptions do not cause a loss of generality in the results discussed in the paper.

Effective demands for product types are computed by netting out the available inventory of each item belonging to the product type. Therefore, in our model formulation,  $I_{i0} = 0$  for  $i=1,2,\dots,I$ . For the computation of effective demand the reader is referred to [1].

Problem (P) is solved with a rolling horizon of length  $T$ . At the end of each time period, new information becomes available and is used to update the model. Only the results pertaining to the first  $L+1$  periods for product types, and the first period for part types are implemented. Constraints (2.2) couple part type requirements and product type production. The other set of constraints involve either part types or product types, but not both. The first  $L$  constraints in (2.1) are included in order to take into consideration the revised forecasts made at the beginning of each period. Although the corresponding parts are already being manufactured, or have already been ordered, minor variations can be absorbed by either expediting the part production or by having a supplier make a special delivery.

To simplify the formulation of Problem (P), we have intentionally omitted planned backorders, hiring and firing, lost sales, and subcontracting. If needed, these can easily be incorporated.

A critical point in the two-stage model is the definition of the parameters  $f_{jk}$ . Theorem 2.1 below shows how those parameters are computed. It also demonstrates that, under certain hypotheses, the definition adopted implies the existence of a feasible disaggregation scheme.

Let  $j$  denote a generic product family of product type  $i$ , let  $n$  be a

generic part in part type k, and define  $f_{ik}$  as a weighted average of the  $f_{ijkn}$  as:

$$f_{ik} = \frac{\sum_{j \in J(i)} \sum_{n \in N(k)} \bar{d}_j f_{ijkn}}{\sum_{j \in J(i)} \bar{d}_j} \quad i=1,2,\dots,I; k=1,2,\dots,K \quad (2.3)$$

where  $J(i)$  is the set of indices of the product families in product type  $i$ ,  $N(k)$  is the set of indices of parts in part type  $k$ ,  $\bar{d}_j$  is the annual demand of family  $j$ , and  $f_{ijkn}$  is the number of units of part  $n$  required by each unit of product family  $j$ . Note that  $f_{ijkn}$  is well defined since, by definition, the families in a product type require the same number of units of the same parts.

It is important to realize that the parameter  $f_{ik}$  represents a weighted average of the parts required by individual items. Thus the solution of the aggregate Problem (P), does not assure the existence of feasible disaggregation even with perfect forecasts. Fortunately, under mild conditions feasibility can be achieved as is shown in the following theorem.

Theorem 2.1: Assume that a perfect forecast is available, the initial inventory of every product family is equal to zero, and that Problem (P) is solved just once (i.e., it is not solved on a rolling horizon basis). The first  $L$  constraints in (2.1) are deleted. Then, the initial inventory of part type  $k$  plus the production scheduled by Problem (P) for this part type up to period  $\tau$  is sufficient to satisfy the sum of the demands, corresponding to the interval  $[1, \tau]$ , of all parts in part type  $k$  for every  $\tau$ , such that,  $1 \leq \tau \leq T-L$ .

Proof: Denote  $m_i(R_{it+L} + O_{it+L})$  by  $X_{it+L}$ . The production of part type  $k$  from periods 1 to  $\tau$  plus its initial inventory, for a generic  $\tau$  in the interval  $[1, T-L]$  is:

$$\begin{aligned} \hat{I}_{ko} + \sum_{t=1}^{\tau} \hat{m}_k (\hat{R}_{kt} + \hat{O}_{kt}) &= \hat{I}_{k\tau} + \sum_{t=1}^{\tau} \sum_{i=1}^I f_{ik} X_{it+L} = \\ &= \hat{I}_{k\tau} + E_k \end{aligned} \quad (2.4)$$

where  $E_k = \sum_{t=1}^{\tau} \sum_{i=1}^I f_{ik} X_{it+L}$ , and the first equality follows from (2.2).

The sum of the demands of all parts, corresponding to the interval  $[1, \tau]$ , in part type  $k$  is:

$$\sum_{i=1}^I \sum_{t=1}^{\tau} \sum_{j \in J(i)} \sum_{n \in N(k)} f_{ijn} \bar{d}_{jt+L} = B_k \quad (2.5)$$

where  $\bar{d}_{jt+L}$  denotes the demand of product family  $j$  in period  $t+L$ . Recall that, by assumption, the initial inventories of each finished product family is zero.

We first show that  $E_k \geq B_k$ . Since all items in a given product type  $i$  have the same seasonality it follows that the ratio of cumulative family demands within a product type remains constant:

$$\frac{\sum_{t=1}^{\tau} \bar{d}_{jt+L}}{\sum_{j \in J(i)} \sum_{t=1}^{\tau} \bar{d}_{jt+L}} = \frac{\bar{d}_j}{\sum_{j \in J(i)} \bar{d}_j} \quad (2.6)$$

Hence, from (2.3) and (2.6),

$$f_{ik} = \frac{\sum_{j \in J(i)} \sum_{n \in N(k)} \sum_{t=1}^{\tau} f_{ijn} \bar{d}_{jt+L}}{\sum_{j \in J(i)} \sum_{t=1}^{\tau} \bar{d}_{jt+L}},$$

and

$$\sum_{j \in J(i)} \sum_{t=1}^{\tau} f_{ik} \bar{d}_{jt+L} = \sum_{j \in J(i)} \sum_{n \in N(k)} \sum_{t=1}^{\tau} f_{ijn} \bar{d}_{jt+L},$$

or, equivalently, since  $\bar{d}_{it+L} = \sum_{j \in J(i)} \bar{d}_{jt+L}$  we have

$$\sum_{i=1}^I \sum_{t=1}^T f_{ik} \bar{d}_{it+L} = \sum_{i=1}^I \sum_{j \in J(i)} \sum_{n \in N(k)} \sum_{t=1}^T f_{ijn} \bar{d}_{jt+L} . \quad (2.7)$$

The aggregate Problem (P) implies that

$$\sum_{t=1}^T x_{it+L} \geq \sum_{t=1}^T \bar{d}_{it+L} . \quad (2.8)$$

Therefore, by substituting (2.8) in (2.7) we obtain

$$E_k \geq B_k . \quad (2.9)$$

Since  $\hat{I}_{kT} \geq 0$ , it follows by (2.4), (2.5), and (2.9) that

$$\hat{I}_{ko} + \sum_{t=1}^T \hat{m}_k (\hat{R}_{kt} + \hat{O}_{kt}) \geq \sum_{i=1}^I \sum_{j \in J(i)} \sum_{n \in N(k)} \sum_{t=1}^T f_{ijn} \bar{d}_{jt+L} .$$

Despite the fact that Theorem 2.1 establishes feasibility conditions for the case of no forecast error and a stationary horizon, our computational experience indicates that good results are obtained even in the absence of these conditions. There are two reasons to support this fact. First, in our computational work, we have observed that the rolling production schedules obtained are quite stable. Second, it can be shown that if the initial inventories are nonzero, but the product families' initial inventories are well balanced, i.e., if

$$\frac{\sum_{j \in J(i)} \bar{I}_{jo}}{\sum_{j \in J(i)} \bar{I}_{jo}} = \frac{\sum_{j \in J(i)} \bar{d}_j}{\sum_{j \in J(i)} \bar{d}_j} , \quad \text{for } i=1,2,\dots,I \text{ and } j \in J(i) \quad (2.10)$$

then, Theorem 2.1 holds.

We conclude this section by pointing out that the setup costs have been intentionally ignored in Problem (P). They are considered during the disaggregation process. This procedure is acceptable whenever setup costs are not a significant portion of total costs. When this is not the case,



a procedure similar to the one devised in [1] for adjusting the hierarchical method for high setup costs can be developed.

### 3. TWO-STAGE HIERARCHICAL PLANNING SYSTEM - THE DISAGGREGATION PROCEDURE

The disaggregation of the solution of the aggregate Problem (P) is performed in two steps. Initially, product family requirements over the first  $L+1$  periods and part requirements for period 1 are determined. In the second step, the production quantities for product items in period 1 are obtained by disaggregating the corresponding production quantities of the product families to which they belong. Let  $X_{it}$  and  $\hat{X}_{kt}$  denote, respectively,  $m_i(R_{it} + O_{it})$  and  $\hat{m}_k(\hat{R}_{kt} + \hat{O}_{kt})$ .

#### Step 1: Product Families and Part Requirements

To determine the production quantities for product families and parts, the following problem needs to be solved:

Problem (PD)

$$\text{minimize } \sum_{t=1}^{L+1} \sum_{i=1}^I \sum_{j \in J(i,t)} (s_{jnt} / Q_{jt}) + \sum_{k=1}^K \sum_{n \in N(k,1)} (\hat{s}_{knl} / \hat{Q}_{nl}) \quad (3.1)$$

$$\text{subject to } \sum_{i=1}^I \sum_{j \in J(i,t)} f_{ijkn} Q_{jt} \leq \hat{a}_{nt} - \hat{s}_{nt} \quad k=1, \dots, K; n \in N(k); t=1, \dots, L \quad (3.2)$$

$$\sum_{i=1}^I \sum_{j \in J(i,L+1)} f_{ijkn} Q_{jL+1} \leq \hat{a}_{nL+1} - \hat{s}_{nL+1} + \hat{Q}_{nl} \quad k=1, \dots, K; n \in N(k) \quad (3.3)$$

$$\sum_{j \in J(i,t)} Q_{jt} = X_{it} \quad i=1, \dots, I; t=1, \dots, L+1 \quad (3.4)$$

$$\sum_{n \in N(k,1)} \hat{Q}_{nl} = \hat{X}_{kt} \quad k=1, \dots, K \quad (3.5)$$

$$l_{b_{jt}} \leq Q_{jt} \leq ub_{jt} \quad i=1, \dots, I; j \in J(i,t); t=1, \dots, L+1 \quad (3.6)$$

$$l_{b_{nl}} \leq \hat{Q}_{nl} \leq \hat{ub}_{nl} \quad k=1, \dots, K; n \in N(k,1) \quad (3.7)$$

The setup cost of product family  $j$  is denoted by  $s_j$ . The demand of family  $j$  over the run-out time of the product type to which it belongs is represented by  $D_{jt}$ . The run-out time is the number of time periods in which the available inventory plus the production quantity  $X_{jt}$  minus the safety stock of the product type is expected to last. The concept of run-out time has been discussed in [1] and [6]. The lower and upper bounds  $lb_{jt}$  and  $ub_{jt}$  are defined as follows:

$$lb_{jt} = \max(0, d_{jt} - ai_{jt} - ss_{jt}) \quad (3.8)$$

$$ub_{jt} = \max(0, os_{jt} - ai_{jt}) \quad (3.9)$$

where  $d_{jt}$ ,  $ai_{jt}$ , and  $ss_{jt}$  are, respectively, the demand, the available inventory, and the safety stock in period  $t$ . The variable  $Q_{jt}$  denotes the production quantity of family  $j$  in period  $t$ . The parameters and variables with a " $\wedge$ " have the same meaning for parts instead of product families.  $\hat{ai}_{nt}$  includes the number of units of part  $n$  on order or being fabricated that will become available in period  $t$ .  $J(i,t)$  and  $N(k,t)$  are, respectively, the set of indices of families in product type  $i$  and the set of indices of parts in part type  $k$  that have a strictly positive lower bound in period  $t$ . These are the families and parts that are triggered in period  $t$ .

The objective function (3.1) assumes that the family run quantities are proportional to the setup costs and to the annual demand for a given family. This assumption, which is the basis of the economic order quantity formulation, tends to minimize the average annual setup cost. Notice that the demand terms in the objective function,  $D_{jt}$  and  $\hat{D}_{kl}$  cover a planning horizon equal to the run-out time. This is consistent with the myopic rules developed in [1]. Constraint (3.2) could have been omitted and the productions corresponding to the lead time "frozen". However, we have

chosen not to freeze the production over the horizon  $L$  since some corrections can be accommodated in practice either by expediting production or by having special deliveries made by suppliers. Problem (PD) has a convex objective function with linear constraints. It has been solved by the Frank and Wolfe algorithm [4].

Theorem 3.1 below shows that under certain conditions a disaggregation problem, intimately related to Problem (PD), can be decomposed into continuous convex knapsack subproblems of the same type as those that arise in the single-stage hierarchical model developed in [1] and [2]. The advantage is that the knapsack problems can be solved by an efficient algorithm reported in [3].

The lower and upper bounds (3.8) and (3.9) for  $t=2, \dots, L+1$  are a function of the disaggregation procedures used in periods 1 to  $L$ , since the available inventories are affected by those disaggregations. However, because the rolling horizon production schedules tend to be quite stable, it is possible to estimate the lower and upper bounds using the results obtained from the disaggregation of the last aggregate schedule available. We denote these estimates by  $lbe_{jt}$  and  $ube_{jt}$  and rewrite (3.6) as

$$\left. \begin{aligned} lb_{j1} &\leq Q_{j1} \leq ub_{j1} & i=1, \dots, I; j \in J(i, 1) \\ lbe_{jt} &\leq Q_{jt} \leq ube_{jt} & i=1, \dots, I; j \in J(i, t); \\ & & t=2, \dots, L+1 \end{aligned} \right\} (3.6)'$$

Problem (PD) with (3.6)' instead of (3.6) is denoted by Problem (PDe).

**Theorem 3.1:** Assume that a perfect forecast is available for the first  $L+1$  periods, and that the product families in each product type require the same number of units of each part, i.e.,  $f_{ijkn} = q_{ikn}$  for  $i=1, 2, \dots, I$ ;  $j \in J(i)$ ,  $k=1, \dots, K$ , and  $n \in N(k)$ . Then Problem (PDe) can be decomposed in  $[I(L+1)+k]T$  continuous convex knapsack problems.

Proof: Due to the assumption of perfect forecasts, safety stocks can be deleted and constraints (3.2) and (3.4) become:

$$\sum_{i=1}^I \sum_{j \in J(i,t)} f_{ijkn} Q_{jt} = \sum_{i=1}^I q_{ikn} X_{it} \leq \hat{a}_{nt} \quad k=1, \dots, K; n \in N(k); t=1, \dots, L \quad (3.10)$$

and

$$\begin{aligned} \sum_{i=1}^I \sum_{j \in J(i, L+1)} f_{ijkn} Q_{jL+1} &= \sum_{i=1}^I q_{ikn} X_{iL+1} \leq \\ &\leq \hat{a}_{nL+1} + \hat{Q}_{n1} \quad k=1, \dots, K; n \in N(k) \end{aligned} \quad (3.11)$$

where the first equalities in (3.10) and (3.11) follow from (3.4). The terms in the left-hand sides of the inequalities in (3.10) and (3.11) are known from the aggregate schedule obtained from Problem (P). Moreover,

$$\hat{a}_{nt+1} = \hat{a}_{nt} - \sum_{i=1}^I q_{ikn} X_{it} \quad t=1, 2, \dots, L$$

Hence, all terms in the right-hand-side of (3.10) and  $\hat{a}_{nL+1}$  are not dependent on the method of disaggregation. Consequently, constraints (3.10) can be deleted. Therefore, Problem (PDe) can be rewritten as:

Problem (PDe)

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{L+1} \sum_{i=1}^I \sum_{j \in J(i,t)} (s_j D_{jt} / Q_{jt}) + \sum_{k=1}^K \sum_{n \in N(k,1)} (\hat{s}_k \hat{D}_{kl} / \hat{Q}_{n,1}) \\ \text{subject to} \quad & \sum_{i=1}^I q_{ikn} X_{iL+1} \leq \hat{a}_{nL+1} + \hat{Q}_{n1} \quad k=1, \dots, K; n \in N(k) \\ & \sum_{j \in J(i,t)} Q_{jt} = X_{it} \quad i=1, \dots, I; t=1, \dots, L+1 \\ & \sum_{n \in N(k,1)} \hat{Q}_{n1} = \hat{X}_{k1} \quad k=1, \dots, K \\ & lb_{j1} \leq Q_{j1} \leq ub_{j1} \quad i=1, \dots, I; j \in J(i,1) \end{aligned}$$

$$lbe_{jt} \leq Q_{jt} \leq ube_{jt} \quad \begin{matrix} i=1, \dots, I; j \in J(i, t); \\ t=2, \dots, L+1 \end{matrix}$$

$$\hat{l}b_{n1} \leq \hat{Q}_{n1} \leq \hat{ub}_{n1} \quad k=1, \dots, K; n \in N(k, 1)$$

Problem (PDe) can be decomposed in the following  $[I(L+1)+K]$  continuous convex knapsack problems:

Problem [PK(i, 1)]

For  $i=1, 2, \dots, I$

$$\text{minimize} \quad \sum_{j \in J(i, 1)} s_j^D / Q_{j1}$$

$$\text{subject to} \quad \sum_{j \in J(i, 1)} Q_{j1} = X_{i1}$$

$$lb_{j1} \leq Q_{j1} \leq ub_{j1} \quad j \in J(i, 1)$$

Problem [PK(i, t)]

For  $i=1, 2, \dots, I$  and  $t=2, \dots, L+1$

$$\text{minimize} \quad \sum_{j \in J(i, t)} s_j^B / Q_{jt}$$

$$\text{subject to} \quad \sum_{j \in J(i, t)} Q_{jt} = X_{it}$$

$$lbe_{jt} \leq Q_{jt} \leq ube_{jt} \quad j \in J(i, t)$$

Problem [PD(k)]

For  $k=1, 2, \dots, K$

$$\text{minimize} \quad \sum_{n \in N(k, 1)} \hat{s}_k^D \hat{D}_{kl} / \hat{Q}_{n1}$$

$$\text{subject to} \quad \sum_{n \in N(k, 1)} \hat{Q}_{n1} = \hat{X}_{k1}$$

$$\text{maximum} \quad (\hat{l}b_{n1}; \sum_{i=1}^I q_{ikn} X_{iL+1} - \hat{a}_{nL+1}) \leq \hat{Q}_{n1} \leq \hat{ub}_{n1}$$

$$n \in N(k, 1)$$

The assumption of perfect forecast during the first  $L+1$  periods allows us to decompose the problem into a set of knapsack problems that can be solved quite easily. Even if this assumption does not strictly hold, we can expect that moderate forecast errors can be absorbed by appropriate levels of safety stocks and the problem can still be decomposed without severe departures from optimality.

### Step 2: Product Item Requirements

The last production quantities to be determined are the number of units of each product item to be assembled in the present period. That is, we need to disaggregate the quantities  $Q_{jl}$  obtained from either Problem (PD) or from Problem [PK(i,1)]. The disaggregation is performed through the following knapsack problem for each family  $j$  in  $J(i,1)$ ,  $i=1,2,\dots,I$ .

$$\begin{aligned} &\text{Problem [F(j,1)]} \\ &\text{minimize } \sum_{v \in V(j)} \left[ \frac{Q_{jl} + \sum_{v \in V(j)} (a_{i_{vl}} - ss_{vl})}{\sum_{v \in V(j)} d_{vl}} - \frac{z_{vl} + a_{i_{vl}} - ss_{vl}}{d_{vl}} \right]^2 \\ &\text{subject to } \sum_{v \in V(j)} z_{vl} = Q_{jl} \\ &\quad lb_{vl} \leq z_{vl} \leq ub_{vl} \quad v \in V(j) \end{aligned}$$

The parameters  $d_{vl}$ ,  $a_{i_{vl}}$ ,  $ss_{vl}$ ,  $lb_{vl}$ , and  $ub_{vl}$  denote, respectively, the demand, available inventory, safety stock, lower bound and upper bound of product item  $v$  in time period  $l$ .  $V(j)$  is set of indices of the product items in product family  $j$ . The lower and upper bounds  $lb_{vl}$  and  $ub_{vl}$  are computed in the same way as in (3.8) and (3.9). The objective function in Problem [F(j,1)] attempts to equalize the run-out time of each item in family  $j$  and the run-out time of family  $j$ . Problem [F(j,1)] is a continuous convex knapsack problem with bounded variables. An effective algorithm to solve it has been provided in [3].

#### 4. COMPUTATIONAL RESULTS - COMPARING HIERARCHICAL PRODUCTION PLANNING WITH MATERIAL REQUIREMENTS PLANNING (MRP)

In this section we report the results of extensive computational experiments conducted to assess the performance of the proposed two-stage hierarchical planning system. Since MRP is so widely used to deal with production planning and inventory control in multi-stage settings, we have contrasted the two approaches.

##### 4.1 The MRP Model Tested

The essential features of MRP are illustrated in Figure 4.1. Although we will not attempt to present a comprehensive coverage of this subject, it is important to emphasize the framework of MRP so as to facilitate a proper comparison between that methodology and the HPP system.

First, MRP defines the production quantities for individual product items through a planning horizon at least equal to the maximum lead time of the parts needed for the assembly of finished products. This production plan becomes the master schedule.

It is worth noting that in many MRP systems in use, the master schedule is viewed as an external input. The lack of an appropriate support for managers to generate a good master schedule has been cited as one of the major weaknesses of MRP [12]. Extensive research is being undertaken to address that weakness today.

The master schedule of production item quantities is then exploded to compute requirements for all the part items. When all the requirements for a given part item have been consolidated, an individual production schedule is developed for each item.

There are many alternative procedures for calculating the part item production quantities. Ref. [12] reviews 10 such approaches.



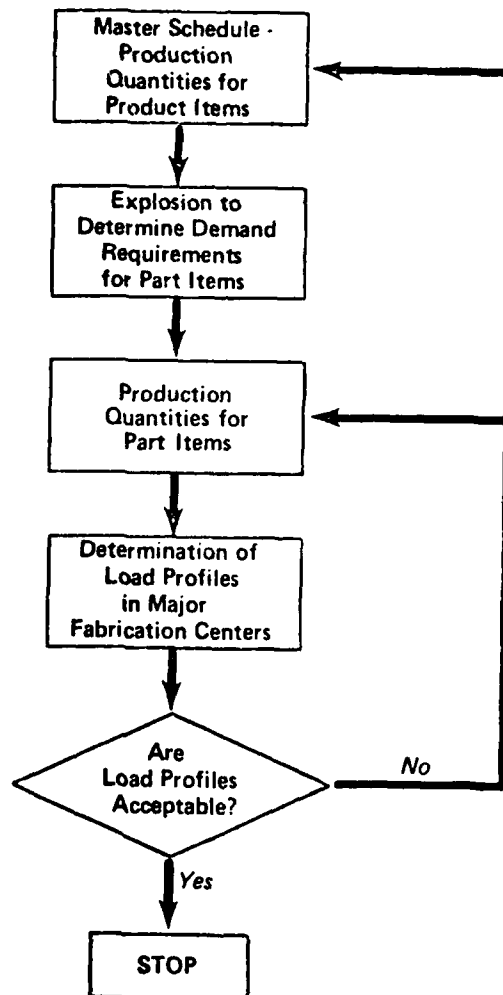


Figure 4.1: The Essence of MRP Computations

It is important to realize that at this step no effort has been made to take into account the economics of point part production; in fact, total aggregate parts scheduled for production might result in unacceptable or infeasible work load fluctuations. To correct for these possibilities, MRP calculates the work load profiles by major fabrication center. These profiles are examined by operating managers and, if undesirable patterns are detected, changes are introduced either at the master schedule level, or at the part item scheduling level.

When MRP is conceived within the framework just described, it is, in essence, a simulation tool which allows managers to test some suggested production programs and identify their consequences.

If we contrast MRP with the proposed HPP system, we can identify several areas of fundamental differences. HPP determines a joint product type-part type aggregate schedule. That schedule is both feasible and attempts to optimize primary costs. Moreover, the fact that it is an aggregate schedule clearly facilitates genuine understanding of its implications. The HPP disaggregation process that leads to part item and finished product item scheduling is focused only on the immediate relevant time period. This avoids an excessive amount of data and computational work, because long term consequences have already been accounted for at the aggregate planning level.

Figure 4.2 depicts the way in which we modeled the MRP system for the purpose of experimental comparison. Our primary concern was to implement computational rules for MRP that could provide unbiased grounds for evaluation. The master schedule was determined by disaggregating the solution of an aggregate linear programming model for product types for the full planning horizon. The disaggregation procedure used was Equalization-of-Run-Out Time. As discussed in [1], this disaggregation rule allows for

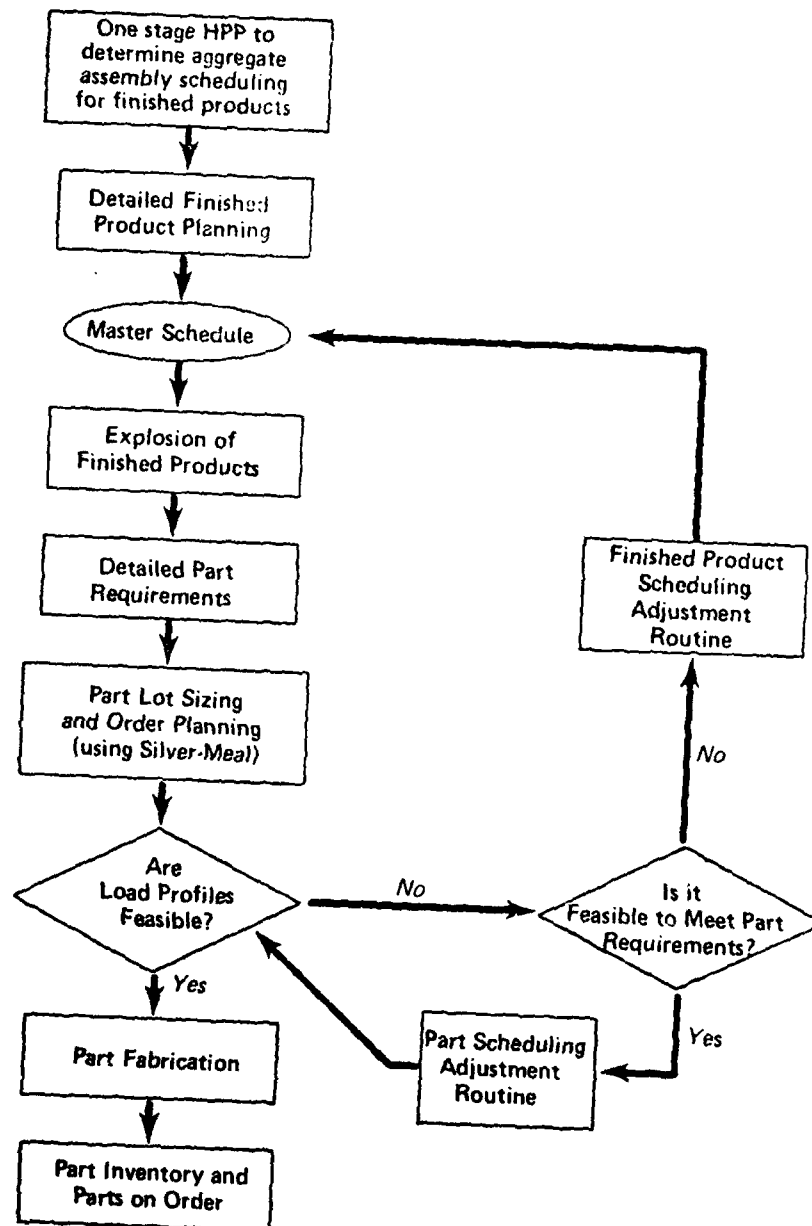


Figure 4.2: "The MRP" method used for comparative purposes

an effective and consistent detailed production schedule when the full planning horizon is involved.

After the individual product item quantities were exploded to determine part item requirements, the Silver-Meal algorithm was used to determine part item production schedules. As reported in Peterson and Silver [11], that method performs quite effectively when compared with alternative lot sizing methodologies.

The last element that we introduced in modeling MRP was the ability to correct for unattractive or infeasible production schedules. The consolidated production schedule for part items was examined and adjusted.

For the sake of brevity, we are referring to that system as "the MRP system". The reader should recognize that it is not in the spirit of our work to make definite comments on another methodology. This would have been an impossible task since MRP can be viewed as a broad approach to production planning rather than a specific, well-defined methodology.

In the remaining part of this section we will present the product structure used in the comparison, and the computational results comparing HPP with the MRP system outlined above.

#### 4.2 Product and Part Structure

The data used in the experiments were scaled-down from a pencil manufacturing company. This firm assembles a variety of pencils requiring a number of distinct components which are shown in Table 4.1. Individual pencils are the product items. Identical-sized pencils were grouped into a single product type, requiring similar assembly times. Within a product type, items sharing common setup costs and having the same inscriptions were grouped into product families. Parts sharing common production facilities were classified into part types. This resulted in three part types:

Table 4.1: The Finished Product/Part Structure

The Product — Pencils

Variations — Sizes - Product Types  
Insignias on the Side - Product Families  
Colors - Items

The Parts — Wood  
Lead  
Erasers (not all pencils require this)

THE NUMBER OF PARTS REQUIRED PER UNITS OF FINISHED PRODUCT

		<u>PART TYPE 1</u>	<u>PART TYPE 2</u>	<u>PART TYPE 3</u>	
<u>FINISHED PRODUCT/PART</u>		<u>WOOD</u>	<u>ERASER</u>	<u>LEAD 1</u>	<u>LEAD 2</u>
<u>PENCIL</u>					
Size 1 (Product Type 1)					
Insignia 1 (Family 1)	Color 1	1	1	1	0
	Color 2	1	1	1	0
Insignia 2 (Family 2)	Color 1	1	1	0	1
	Color 2	1	1	0	1
Size 2 (Product Type 2)					
Insignia 1 (Family 1)	Color 1	1	1	1	0
	Color 2	1	1	1	1
Insignia 2 (Family 2)	Color 1	1	1	1	0
	Color 2	1	1	1	0
Insignia 3 (Family 3)	Color 1	1	0	0	1
	Color 2	1	0	0	1

wood, erasers, and lead (this last part type was composed of two part items, lead 1 and lead 2).

In order to test the robustness of the HPP and MRP methodologies, a number of critical parameters were selected to be varied within appropriate ranges. The parameters selected were: finished product capacity, part capacity, forecast errors, seasonality of demands, finished product setup costs, part setup costs, part holding costs, seasonality of finished product demand, and overtime costs. Appendix 1 describes the details of the values tested for each parameter and their corresponding measurements. The total combinations of the values of the parameters tested results in one hundred different runs.

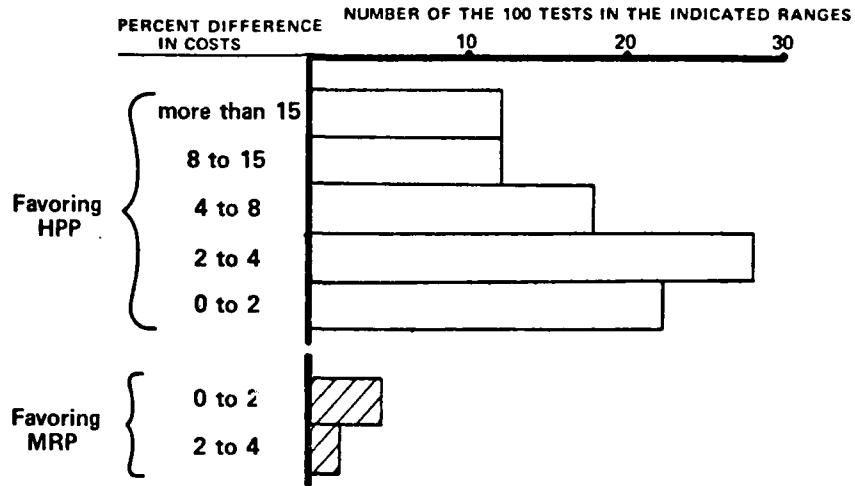
#### 4.3 Computational Results

We compared the performance of MRP and HPP, in terms of costs and backorders. When total annual costs were used as a measure of performance, 93% of the tests favored hierarchical production planning. The maximum cash advantage in these cases was 144%. In those tests favoring MRP, the maximum cost advantage was 3%.

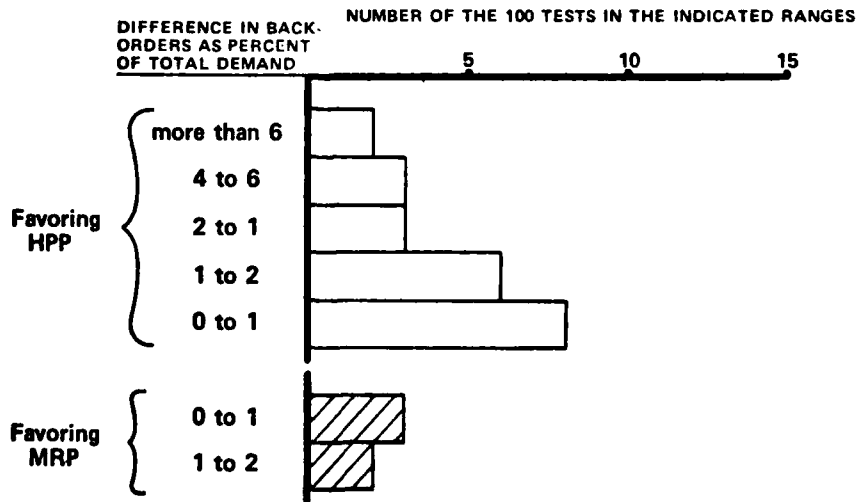
When total backorders as a percent of annual demand were used as the measure of performance, 22% of the tests favored hierarchical production planning, 5% of the tests favored MRP, and the remaining 73% resulted in no backorders under either methodology. In the 5% of the tests favoring MRP, the maximum difference in backorders was 2% of annual demand. In the 22% of the tests favoring hierarchical production planning, the maximum difference was 8%. A summary of the differences in methods for all one hundred tests simulated is illustrated in Figure 4.3.

For the purpose of a more complete comparison, we attempted to identify distinguishing features of the tests which favored MRP and those which most

**THE PERCENTAGE DIFFERENCE IN TOTAL COSTS OF THE TWO METHODOLOGIES**



**THE DIFFERENCE IN TOTAL BACKORDERS AS A PERCENTAGE OF TOTAL ANNUAL DEMAND**



**Figure 4.3: A Summary of the Results Comparing the Two-Stage Hierarchical Model with MRP**

strongly favored the hierarchical approach.

Of the seven tests which favored MRP in terms of costs, none favored MRP in terms of backorders. In two tests, the MRP approach had higher backorders than the hierarchical approach. Of the remaining five tests, two had exceedingly high part capacity, zero forecast error, and very high part-setup costs. The remaining three tests which favored MRP all did so by small amounts and all had medium forecast errors with a positive bias, very high part-setup costs, and high part capacity. This indicates that the MRP approach outperforms HPP in terms of costs only if 1) part capacity is unlimited and forecast error is low, or (2) if part capacity is loose, and finished product forecasts are always high and part setup costs are steep.

The five tests that favored MRP in terms of backorders strongly favored hierarchical production planning in terms of cost. The average difference in total annual cost was 68%. Simultaneously, they all had high seasonality, low finished product capacity, very high part setup costs, and medium or high part capacity. This indicates that if seasonality is high, capacity relatively tight, part setup costs very high, and backorders very costly, the MRP approach is preferable to HPP.

In examining the 11 tests in which the total costs associated with the HPP method were at least 15% smaller than those associated with MRP, no overall conclusions regarding point characteristics can be drawn. It is worth noting that none of the 11 tests had either very high part capacity or very strong positive biases in the forecasts.

In the two cases in which the backorders associated with MRP were greater than the backorders associated with HPP by more than 6% of annual demand, part capacity was low, forecast errors were unbiased, and seasonality was high.



In general, when parts are fabricated internally, the HPP approach appears superior to the MRP method of planning in all but the very unusual cases. This finding is further supported by comparing these two approaches with the Wilcoxon Statistic [9]. This test indicates that the probability that the difference in total cost of the HPP and the MRP approaches comes from a distribution centered at zero is less than .0001 (the normalized Wilcoxon statistics is greater than  $7\frac{1}{2}$  standard deviations). The Statistic supports our conclusion that the HPP methodology is superior to MRP for the cases tested.

## 5. CONCLUSIONS

The HPP system proposed in this paper represents a novel and systematic way to deal with complex production planning decisions faced in a two-stage manufacturing environment. When tested in a relatively simple setting, it provided encouraging results which lead us to believe that the methodology has interesting potentials that could be exploited. In particular, HPP was clearly superior when compared directly with an MRP system.

Given the emerging emphasis on MRP as a planning tool to coordinate fabrication and assembly operations, we feel that serious attention should be focused on hierarchical production planning systems as a topic of future research.

## Acknowledgement

The authors are grateful to Josep Valor for his help in the computational experiments.

#### APPENDIX 1: THE DATA BASE

Given the large number of possible input parameters, we varied those we felt were most important to test and held constant the finished product holding cost, annual demand, the product structure, and productivity. All variables held constant were set to values representative of the actual pencil manufacturer used as a base for our study.

We did not consider it necessary to vary both the finished product and part holding costs, since it is their relative magnitudes that are of primary importance for the purpose of our simulation.

The product/part structure we chose to use throughout the simulation was discussed in section 4 and illustrated in Table 4.1.

Productivity was not varied in the tests simulated. The effects generated by altering productivity are equivalent to those observed when changing capacity. They influence the number of finished products and parts that the system can produce at any moment in time. We felt it unnecessary to vary both parameters.

The measure used for capacity was the minimum fraction of demand that can be satisfied with regular time. The capacity was varied between 20% and 100% for finished products and 20% and 250% for parts and was measured in the following manner.

- (a) Determine the period where the highest average demand per period occurs (based on cumulative demand - see Figure A1). Let us call this period N.
- (b) At period N, compute the average demand per period, or:

$$\text{Average Demand}_N = (\text{Cumulative Demand up to period N})/N$$

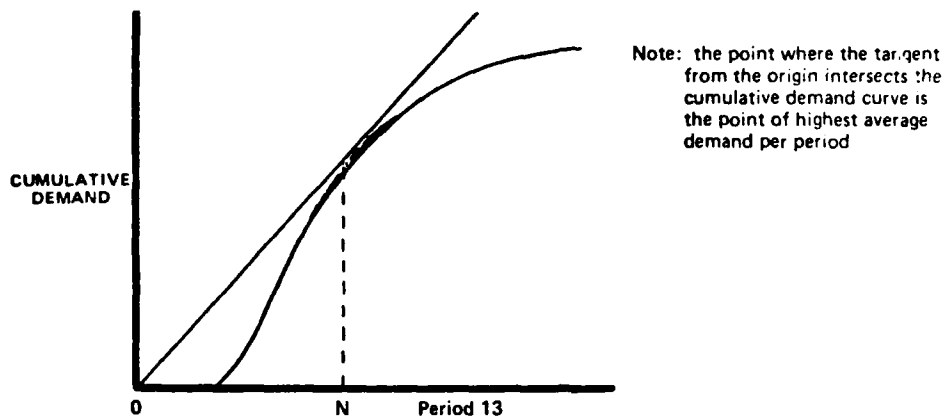


Figure A1

- (c) If capacity with no overtime is equal to Average Demand<sub>N</sub>, then capacity is set to 100%. If capacity with the maximum allowable overtime is equal to Average Demand, then capacity is set at 0%. At all points between these two extremes, capacity is scaled appropriately.

To define high setup costs, we relied upon the impact of those costs on the length of the economic order quantities runs. We defined a high (very high) setup cost to be one associated with a run length greater than two (three) periods. Within the production environment in which we conducted these tests, a three period setup cost was high.

Despite the assumption in the HPP approach that setup costs are secondary, we tested points with high part and finished product setup costs in an attempt to identify situations in which the MRP approach outperformed our hierarchical model.

The forecast error, which was expressed as a percentage of demand, was an increasing function of time. The following three scales for forecast error magnitude were used:

$$0,$$
$$0.01 + 0.02t^{1.3},$$
$$\text{and } 0.05 + 0.02t^{1.1},$$

where  $t$  represents time. Given the magnitude of the forecast error, the probability of an error being positive was defined as the positive bias of the forecast and ranged from zero to one in the tests. Exponentially increasing forecast errors correspond to what we have observed in practice.

Each demand pattern was treated as two connected opposite direction sine waves (as represented in Figure A2). Changes in seasonal patterns was controlled by modifying the coefficient of variation of the seasonal factors ranging from 1.3 to 5.2.

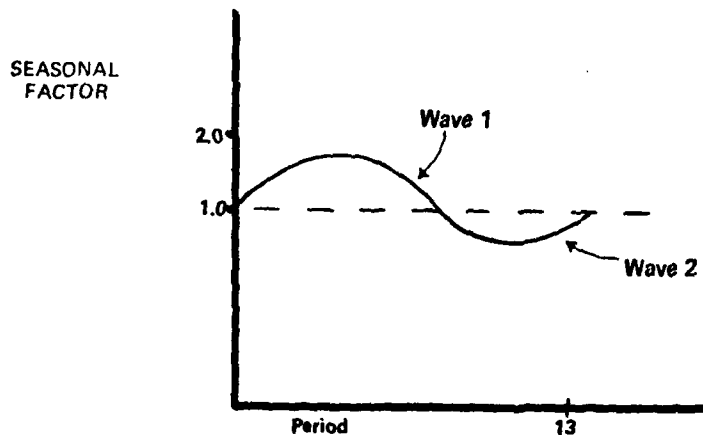


Figure A2

Part holding costs were measured as a percentage of finished product holding costs and varied between 30% and 80%.

Overall, finished product and part capacity and setup costs, part holding costs, forecast errors, and seasonalities assumed a variety of realistic possible values in the tests simulated.

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